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Abstract:

We describe the development of a framework to compute the optimal inventory policy for a large spare parts distribution centre operation in the Refrigeration & Air Conditioning (RA) division of the Danfoss Group in Denmark. The RA division distributes spare parts worldwide for cooling and air-conditioning systems. The warehouse logistics operation is highly automated. However, the procedures for estimating demands and the policies for the inventory control system that were in use at the beginning of the project did not fully match the sophisticated technological standard of the physical system. During the initial phase of the project development we focused on the fitting of suitable demand distributions for spare parts and on the estimation of demand parameters. Demand distributions were chosen from a class of compound renewal distributions. In the next phase, we designed models and algorithmic procedures for determining suitable inventory control variables based on the fitted demand distributions and a service level requirement stated in terms of an order fill rate. Finally, we validated the results of our models against the procedures that had been in use in the company. It was concluded that the new procedures provided a better fit with the actual demand processes and were more consistent with the stated objectives for the distribution centre. We also initiated the implementation and integration of the new procedures into the company's inventory management system.

Keywords: Base-stock policy, compound distribution, fill rate, inventory control, logistics, stochastic processes.

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1. Introduction

In many standard implementations of inventory control systems continuous demand is assumed, and a service level constraint is applied. Lead-time demand is then usually modelled as a (truncated) normally distributed stochastic variable. The service level constraint is most commonly specified either as a fill rate requirement or as a cycle-stock service level requirement. This may work well in many settings, particularly for standard high-volume products. However, in some cases this approach is clearly not satisfactory, especially when demand is lumpy and/or the importance of each customer order is given equal weight. Recently, we have been involved with such a case in our work for the Danfoss Group, one of the largest industrial companies based in Denmark.

The company is involved with research and development, production, sales and service of mechanical and electronic products and controls for several industries. The Danfoss Group is a leading manufacturer of valves and fluid handling components for heating, ventilation and air conditioning, and for industrial applications. It has three main business areas: Refrigeration and Air Conditioning (RA), Heating and Water, and Motion Controls. The company's annual turnover in 2005 was approx. 2,200 million EUR, and it employs about 18,200 people worldwide. The RA division, in which this case study was carried out, accounts for approximately 50% of the total turnover.

At its major distribution centre located in the south of Denmark, the RA division was experiencing typical inventory problems of stock allocation and control, i.e., frequent shortages of certain stock keeping units (SKUs) coupled with a relatively high aggregate level of capital tied up in inventory. The SKUs handled by the distribution centre cover a fairly broad spectrum of spare parts, including many with low frequency but lumpy demand. Initially, we identified two important reasons for the stock control problems. First, the demand distribution used for inventory control purposes did not fit well with the empirical demands. Second, the inventory control method applied was not consistent with the performance criterion used. The first reason is related to the forecasting system, while the second reason is related to the design of the inventory control system. Preferably, these two systems should also be integrated in the total control system. Empirical relevance, consistency of the approach and integration of forecasting and inventory control systems then formed the overall objectives for this case study in order to improve the operations of the RA division's major distribution centre. In the long run, the goal is to obtain similar improvements also at the division's other distribution centres worldwide.

In the first phase of the case study, inspired by the inventory control literature on lumpy demand, we focused on compound Poisson distributions to model the empirical demand distributions. In order to obtain a reasonably good fit, the demand data available first had to undergo a simple filtering process. The choice of compounding distribution was also an issue. Starting with the geometric distribution, we identified a menu of discrete compounding distributions to choose from depending on the empirical data estimates at hand for a particular SKU. In a later phase of the case study we also observed that the Poisson process was not a good description of the demand process. A better match could be obtained by assuming, more generally, a renewal process, and then empirically fitting Erlang-k processes to those SKUs whose demand processes exhibited a more regular pattern.

As regards the performance criteria for the inventory control system, capital tied up in inventory was obviously an important criterion recognized by the company. However, rather than adopting, as in a standard setting, a fill rate or a cycle service level as the service level requirement, the inventory managers argued that an *order fill rate* should be applied. In fact, inventory service performance was already based on measuring the percentage of complete orders for single SKUs filled on time. To distinguish the order fill rate from the standard unit-based fill rate, we term the latter the volume fill rate to indicate its unit-based nature. The basis for using an order fill rate is the focus on individual customer orders and the distribution centre's ability to fill each of those orders in full from inventory. The focus on customer order fulfilment indicates that equal weight is attached to each order irrespective of its size or volume. This corresponds to a cost of shortages related to the occurrence of a shortage, not its size. Of course, if each customer order were for a single unit, then the order fill rate and the volume fill rate would be identical. However, as noted above, this is not always the case, and a customer order may be for several units at a time. Hence, this part of the case study involved development of specifications and algorithms related to the order fill rate in order to harmonize with the

RA division's base-stock inventory control system. Obviously, the specifications and algorithms subsequently also had to be integrated with the empirical demand estimates referred to above.

The remainder of this paper is structured as follows. In Section 2 we give a brief review of some of the relevant literature. Section 3 describes how demand information is used to specify the probability distributions used in the inventory control system. The inventory control system, in particular the specification of base-stock levels in terms of the service level requirement, is outlined in Section 4. In Section 5 we present some data regarding evaluation of the proposed inventory control system. This is followed by a discussion of various implementation and application issues in Section 6. Finally, in Section 7 we summarize the conclusions on the project at its present stage, and discuss routes for future developments in order to make the system even more apt to satisfy the needs of the company for a functional inventory control system.

2. Literature review

In standard inventory control presentations and applications cumulative lead-time demand is often assumed to be specified by a normal distribution. The gamma distribution is sometimes, as in Burgin (1975) and Tyworth et al. (1996), suggested as an alternative to the (truncated) normal distribution. Alternatively, the negative binomial distribution is suggested to model discrete demand over the lead time. It is well known that this distribution is obtained if demand is generated by a pure Poisson process, and thus is unit-sized, and if lead time is gamma distributed (Hadley and Whitin, 1963). For an overview of several other lead-time demand distributions suggested in the literature see Silver et al. (1998).

SKUs with lumpy and low frequency demand are generally more difficult to forecast and control than SKUs with a stable and steady demand (Willemain et al., 2004). Lumpiness refers to the fact that demands are not always unit-sized, and low frequency is sometimes labelled as intermittent. In particular, demand for spare parts is often observed to be characterized by lumpiness and relatively low frequency. Compound Poisson demand processes have been suggested in the inventory literature to model demand with these characteristics.

One of the early contributions on the application of this type of demand process in inventory models is Feeney and Sherbrooke (1966). Ward (1978) also uses a compound Poisson demand process in the development of an inventory control model based on a case study. A more recent example is Matheus and Gelders (2000), who develop exact and approximate reorder point calculation methods for an inventory control policy with a fixed replenishment order quantity and compound Poisson distributed demand. The recently updated textbook by Axsäter (2006) contains a fairly extensive treatment of demand represented by a compound Poisson process in inventory control. In particular, with reference to Feller (1966), Axsäter notes that a non-decreasing stochastic process with stationary and mutually independent increments, which is a common assumption for cumulative demand in inventory control theory, may always be represented as a limit of an appropriate sequence of compound Poisson processes (Axsäter, ibid., p. 77). Renewal compound demand processes for inventory control have been treated to a much lesser extent in the literature, as pointed out by Smith and Dekker (1997). They also note that, whereas a base-stock policy is the optimal policy in a standard cost setting under compound Poisson demand (see, e.g., Zipkin, 2000, Ch. 6), this is not true in general for compound renewal demand. Nevertheless, a base-stock policy might be the preferred policy in a practical setting due to its simplicity.

The choice of compounding distribution for a compound demand process poses a model specification problem with implications for demand parameter estimation, hence forecasting. The stuttering Poisson process, i.e., the compound Poisson process with the geometric distribution as the compounding distribution is frequently proposed in the literature. Recently, Johnston et al (2003) found some empirical evidence to support the assumption of using the geometric distribution as the compounding distribution. The geometric distribution is a special case of the negative binomial distribution, which belongs to the family of power series distributions that also includes the binomial, the Poisson, and the logarithmic distributions. Note that if the order arrival process is Poisson and the compounding distribution is logarithmic, then the cumulative lead-time demand follows a negative binomial distribution (Zipkin, ibid.).

Regarding forecasting of intermittent demand for spare parts, see Hua et al. (2007) for a recent overview. Willemain et al. (2004) categorize forecasting methods for lumpy and low frequent demand. Among the categories is Croston's method; See Croston (1972) for the original reference and Johnston and Boylan (1996) for a more recent reference. Underlying Croston's method is the basic idea of decomposition of the demand process into order sizes and order intervals. This corresponds well with the structure of the compound demand processes referred to above. Johnston and Boylan (1996) extend Croston's approach of order size and interval decomposition by adding estimators for the demand variability. They also compare the decomposition approach to a standard Exponentially Weighted Moving Average (EWMA) procedure and conclude that it works better than the EWMA for SKUs with low frequency demand. Hua et al. (2007) suggest forecasting intermittent demand by integrating estimation of an auto-correlated demand process with a logistic regression of non-zero demands. Improvements compared to Croston's method are reported, but the regression procedure tends to be demanding in terms of data requirements. Park (2007) analyzes different approaches to decomposition of compound cumulative demand distributions. In addition to the order size and order intensity/inter-arrival time components, the lead time component is also considered.

Service levels are used in inventory control systems for performance evaluation and in target setting as substitutes for shortage costs that are hard to estimate. A review of standard service level measures and their relationships to shortage costs and different inventory control policies is provided by Schneider (1981). Another overview is provided in Tempelmaier (2000), who considers service level measures from a supply chain perspective. Most textbooks on operations management and/or inventory control include treatments of this topic, because service levels are ubiquitous in practical inventory systems. One of the most commonly used performance measures in inventory control is the (volume) *fill rate*, defined as the fraction of total demand volume that can be satisfied from inventory without shortages (Silver et al., 1998, p. 245). Song (1998) deals with the composite fill rate in a setting with multiple SKUs. Chen and Krass (2001) consider minimal service level constraints.

Somewhat less common as a performance measure in inventory control is the *order fill rate* (*OFR*), specified as the fraction of complete orders that can be filled directly from inventory. Sometimes it is also labelled the *line fill rate* or the (*perfect*) *order fulfilment rate*. The closely related issue of *complete fill rates* is considered by Boylan and Johnston

(1994) in their study of relationships between different *ex post* service level measures. The term *complete fills* is used by Feeney and Sherbrooke (1966). In Larsen and Thorstenson (2007) the *OFR* is specified for a base-stock inventory control system and its relation to the standard (volume) fill rate and to other service measures such as the *ready rate* (see, e.g., Axsäter, 2006) is analyzed.

3. Demand information

Describing the demand process by a probability model is one of the essential inputs in an analytical model of a stochastic inventory control system. Therefore, before developing any kind of mathematical inventory model for practical application, it must be clarified which sort of demand data can be extracted from the company's records of demand information. In particular, is the demand data available in an aggregated form, such that only information about the total demand in, say, a week can be extracted, or is it possible to obtain information about every specific customer order? In this case it turned out that we were fortunate to be able to acquire detailed data of the latter form from the company's records. Hence, it was straightforward to decompose the demand process into two components: the size of an individual customer order (modelled as a positive integer-valued random variable X) and the order interval, the time between two consecutive order requests (modelled as a positive continuous random variable T). The demand information extracted from the company's records then made it possible to fit empirical estimates of the customer order sizes and the order intervals to some alternative theoretical probability distributions for X and T.

Before considering the choice of appropriate probability distributions, we took an issue into account that is not strongly emphasized in standard texts on mathematical models for inventory control, like Silver et al (1998) and Zipkin (2000). It is the issue of *advance-order information*. When a request for an order i is received at time point τ_i, there is often an agreed order delivery lead time DT_i, so that the customer requires a (physical) delivery at time point $\tau_i + DT_i$. Usually, in standard texts, it is tacitly assumed that $DT_i = 0$, implying that customers expect instant delivery. In Silver et al (1998) it is not mentioned at all that DT_i might be positive. In Zipkin (2000) the possibility of a positive DT_i is only mentioned in an exercise (Exercise 6.1). However, to some extent the issue has been treated in the literature; See e.g. Hariharan and Zipkin (1995) and Marklund (2006). The former show how advance order information can be used to improve performance for single-level and serial inventory systems, while the latter generalizes the results to divergent supply chains with non-identical customer classes.

Particularly, if the order delivery lead time DT_i is longer than the estimated replenishment lead time L, then order i is a completely *known order*, because there is enough time to plan for this order without depending on inventory holding. Hence, there is no need to keep an inventory for known orders to avoid the risk of shortage (only to capture possible economies of scale in ordering; however, this possibility was ruled out in this case by the company's choice of a base-stock ordering system for inventory control). To distinguish the known customer orders from the others, we introduced a simple (and crude) filtering rule: All orders with an agreed order delivery lead time longer than the estimated replenishment lead time of the SKU ($DT_i > L$) are excluded from the data. Consequently, it is also implied that the known orders should be handled separately from the stochastic inventory control system.

The remaining data, i.e. data for the (partly) *unknown orders* are subsequently used for estimation of parameters for the random variables X and T. It is of course essential that this filtering process is carried out before parameters of T are estimated. Otherwise, one would exaggerate the intensity of the arrivals of the (unknown) order requests. Concerning the estimation for X, one could argue that if there is no significant difference between the sizes of known and unknown orders, then in order to have a larger data set, one should estimate parameters of X on the basis of all order-size information. However, in some cases we observed that the known order sizes had characteristics that were quite different from the unknown orders. They are often larger and also frequently related to 'box' or package sizes. Because of this possible effect, the parameters for X are also estimated using only the unknown customer order data. As noted above, the filtering rule is crude and any information contained in order delivery lead times $0<DT_i<L$ is currently disregarded. We comment on this issue in Section 6.

3.1 Estimation of the customer order size (*X*)

To estimate the parameters of the customer order-size distribution, we consider an SKU for which - given a historic time window - there is a record of *n* orders of sizes $x_1, x_2, \ldots, x_n$. From this we compute the empirical mean, *EmpM*, and variance, *EmpVar*:

$$EmpM = \frac{1}{n}\sum_{i=1}^{n} x_i$$

and

$$EmpVar = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - EmpM)^2 .$$

These estimates will be used as part of the demand forecast needed to compute the inventory control policy for an SKU. Thus, it is assumed that demand is stationary in the chosen historic time window, and in the near future when the estimates are going to be applied. Hence, any systematic patterns, such as drift, seasonality or trend, in the order size observations are disregarded. We then apply one of three alternative theoretical probability distributions to fit data, namely the (delayed) negative binomial, the (delayed) binomial and the (delayed) Poisson distribution. The term delayed indicates that compared to standard textbooks expositions of these distributions the ones employed here do not contain realizations of zero (demands). This implies no lack of generality as the parameters of a renewal process can easily be adjusted to exclude observation of zeros. The three discrete distributions belong to the family of power series distributions (PSDs). They are motivated by their computational tractability and by the fact that they allow for different levels of the customer order-size variance in relation to its mean. The Poisson distribution is the limiting case of both the negative binomial distribution and the binomial distribution, when the variance approaches the mean. Thus, the PSD family encompasses a broad set of unimodal distributions with appealing shapes.

When *X* follows a *(delayed) negative binomial distribution*, the probabilities $P(X = j)$ are given by

$$P(X = j) = \frac{(s+j-2)!}{(j-1)!(s-1)!}(1-\rho)^s \rho^{j-1} \quad j=1,\ 2, \ldots,$$

where the form parameter s is positive (but does not have to be integer valued), and the probability parameter ρ satisfies $0 < \rho < 1$. The geometric distribution is the special case, when s=1. The mean, $E[X]$, and variance, $V[X]$, of the delayed negative binomial distribution are given by

$$E[X] = \frac{s\rho + 1 - \rho}{1 - \rho}$$

and

$$V[X] = \frac{s\rho}{(1-\rho)^2}.$$

Note that

$$V[X] > E[X] - 1.$$

Therefore, it only makes sense to model customer order size as negative binomially distributed, if

$EmpVar > EmpM - 1.$

When this relation holds for the SKU, the parameters s and ρ are estimated from the two equations:

$$EmpM = \frac{s\rho + 1 - \rho}{1 - \rho},$$

$$EmpVar = \frac{s\rho}{(1-\rho)^2},$$

which have the solution $\hat{s}$ and $\hat{\rho}$ given by

$$\hat{\rho} = \frac{EmpVar - EmpM + 1}{EmpVar},$$

$$\hat{s} = \frac{(1-\hat{\rho})(EmpM - 1)}{\hat{\rho}}.$$

Note also that for the negative binomial distribution

$$P(X = 1) = (1-\hat{\rho})^{\hat{s}} = e^{\hat{s}\ln(1-\hat{\rho})}$$

and

$$P(X = j+1) = \frac{\hat{s} - 1 + j}{j} \hat{\rho} P(X = j) \quad j=1,\ 2, \ldots,$$

so that the probabilities can be computed recursively.

Alternatively, the parameters *ρ* and *s* could have been estimated by the method of '*zeros and mean*'; see Anscombe (1949, 1950). This method also involves solving a system of equations. However, instead of the empirical variance it focuses on the fraction of orders of unit size (as the delayed negative binomial distribution applied here does not include realization of zeros). Anscombe (1949; p. 167) recommends this method for $s < 1$, while he recommends the method used in this paper when $s > 1$. As evidenced by Table 6, our dataset contains SKUs, whose order size distribution is fitted to a (delayed) negative binomial distribution, with both $s < 1$ and $s > 1$. Thus, a possible improvement, but at the expense of more computational effort, would be to incorporate both methods into the estimation procedure.

When *X* follows a *(delayed) binomial distribution*, the probabilities $P(X = j)$ are given by

$$P(X = j) = \frac{(n-1)!}{(j-1)!(n-j)!} p^{j-1} (1-p)^{n-j} \qquad j=1,\ 2, \ldots,\ n\ ,$$

where the parameter n is positive and integer valued, while the parameter p satisfies $0 < p < 1$. Its mean and variance are given by

$E[X] = (n-1)p + 1$

and

$V[X] = (n-1)p(1-p).$

Note that

$V[X] < E[X] - 1$

Therefore, it only makes sense to model customer orders as binomially distributed, if

$EmpVar < EmpM - 1.$

When this relation holds for the SKU, we estimate the parameters n and p by the two equations

$EmpM = (n\text{-}1)p + 1$,

$EmpVar = (n\text{-}1)p(1\text{-}p)$,

which have the solution $\hat{n}$ and $\hat{p}$ given by

$$\hat{p} = \frac{EmpM - 1 - EmpVar}{EmpM - 1},$$

$$\hat{n} = \frac{EmpM - 1}{\hat{p}} + 1 .$$

It is required that $\hat{n}$ is an integer. A simple approach is then to use the solution above rounded up to its nearest integer. Thus

$$\hat{n} = Int(\frac{EmpM - 1}{\hat{p}} + 1.99),$$

where $Int(\cdot)$ returns the integer part of the argument. An alternative and somewhat more sophisticated, but computationally less convenient, approach would be to construct the empirical parameter values from a mixture of two binomial distributions with adjacent values of n; see Bradley and Robinson, 2005, where this approach is applied in a base-stock inventory control model.

Note also that for the (delayed) binomial distribution

$$P(X = 1) = (1 - \hat{p})^{\hat{n}-1} = e^{(\hat{n}-1)\ln(1-\hat{p})}$$

and

$$P(X = j + 1) = \frac{\hat{n} - j}{j} \frac{\hat{p}}{1 - \hat{p}} P(X = j) \quad j=1,\ 2,\ldots,$$

so that the probabilities can again be computed recursively. The recursion formula automatically facilitates that $P(X = j + 1) = 0$, when $j = \hat{n}$.

When X follows a *(delayed) Poisson distribution*, the probabilities $P(X = j)$ are given by

$$P(X=j)=e^{-\delta}\frac{\delta^{j-1}}{(j-1)!}\quad j=1,\ 2, \ldots,$$

where the parameter δ is positive. Its mean and variance are given by

$E[X] = \delta + 1$,

$V[X] = \delta$.

Note that

$V[X] = E[X] - 1$.

Therefore, it only makes sense to model customer order sizes as Poisson distributed if *EmpM - 1* 'is close to' *EmpVar*. Below, it will be explained how the phrase 'is close to' is made operational. When *EmpM - 1* 'is close to' *EmpVar* we estimate the parameter δ by

$\hat{\delta} = EmpM - 1$.

Note further that

$P(X=1)=e^{-\hat{\delta}}$,

$$P(X=j+1)=\frac{\hat{\delta}}{j}P(X=j)\quad j=1,2,\ldots,$$

so that the probabilities can be computed recursively in this case as well.

When *EmpM - 1* 'is close to' *EmpVar*, we have in reality two (or three) distributions to choose between. Therefore, this case can also serve as a test of robustness: Whichever distribution is chosen, the control parameters computed and the corresponding service levels should be almost equal.

To make the choice of probability distribution for the customer order size operational, we introduce a tolerance parameter γ (typically $\gamma = 0.1$). The choice of distribution is then decided by the following rule:

If $EmpVar < (1-\gamma)(EmpM - 1)$ then choose a *delayed binomial* distribution

If $(1-\gamma)(EmpM - 1) \leq EmpVar \leq (1+\gamma)(EmpM - 1)$ then choose a *delayed Poisson* distribution

If $EmpVar > (1+ \gamma)(EmpM -1)$ then choose a *delayed negative binomial* distribution.

By analyzing the same data set with different values of γ, we can implement the robustness test. Setting $\gamma = 0$ eliminates the Poisson distribution as an alternative.

In a later version of the model we made an essential modification to the procedure described above. When fitting to a negative binomial distribution, the fitted parameter value $\hat{\rho}$ sometimes turned out to be very high, around *0.99*. This implies that our model assumed an order-size distribution with a rather large tail probability, i.e. with a rather high probability that a large order will arrive. Consequently, the model proposed a quite large base-stock level. In practice, it was not considered very likely that such a large order should arrive (unexpectedly). Hence, it was felt that the model behaved too conservatively. As a remedy for this an additional input is registered, namely the largest customer order size, *CosMax* (abbreviation for *Customer order size Maximum*).

$CosMax = Max\{x_i: i=1,..,n\}$

The information about *CosMax* is only considered, if the negative binomial distribution is chosen. In that case we first compute the parameter estimates $\hat{s}$ and $\hat{\rho}$ as explained above. Subsequently, these two values are modified to fulfil the following two requirements:

$$EmpM = \frac{s\rho + 1 - \rho}{1 - \rho}$$

and

$P(X \geq CosMax) \leq \varepsilon_1$

where ε_1 is a small number, typically $\varepsilon_1 = 0.01$. By introducing this extra procedure we often manage to correct for the previously observed conservative behaviour of the inventory control model.

3.2 Estimation of the time between consecutive order requests (*T*)

We now turn to estimation of the second component of the demand process, the time between customer orders. Assume that the n orders were recorded in a historic time window of length τ. Analogously to the assumption in Section 3.1, it is assumed here that the pattern of times between customer orders is stationary. The customer order intensity (the number of customer order requests per time unit) is then simply estimated as $\lambda = n/\tau$. In an earlier version of the model, it was assumed that the arrival of order requests could be described by a Poisson process. This is a common assumption in many mathematical models of inventory control (see e.g. Axsäter, 2006 for an overview). However, when examining the actual datasets, this was often observed to be a too restrictive assumption. In reality, the arrival pattern was less erratic than assumed by the Poisson arrival process, which cautiously proposed too high base-stock levels. This effect has also been noted elsewhere in connection with inventory control for spare parts (Smith and Dekker, 1997). Therefore, we decided to collect data for an additional input parameter, *TBOmin* (abbreviation for *minimum Time Between Orders*), which is the minimum time recorded between two successive order requests. This extra input parameter is used to fit the distribution of T to a k-phased Erlang distribution with mean value $1/\lambda$. The phase parameter is then found as the smallest positive value of k satisfying

$$P(T \leq TBOmin;\, k) \leq \varepsilon_2\,,$$

where ε_2 is a small number, typically $\varepsilon_2 = 0.01$. Note that if *TBOmin* is 'small', then the procedure will generate $k = 1$, which implies an exponentially distributed time between orders, i.e. a Poisson process. Thus, this later version of our model is a genuine generalization of the earlier version. The motivation for choosing the Erlang distribution is that it is computationally tractable while being versatile enough to provide a wide variety of shapes.

3.3 Alternative estimation procedures

For some of the simple methods used for estimating the demand process there may exist better alternative procedures. Two alternatives have already been mentioned in Section 3.1 regarding estimation of negative binomially and binomially distributed order sizes. Moreover, instead of solving an equation system based on point estimates, the Maximum Likelihood Estimation (MLE) method might have been applied. However, for the case of the negative binomial distribution it would be more intricate to do MLE, requiring the use of an optimization method; see Law and Kelton (1991; p. 348). We conjecture that the final results obtained by using MLE, and the results from using our simpler method would not differ significantly in practice. Moreover, in many cases the number of observations is rather small, so the result of any (more or less sophisticated) method should be considered with caution.

The introduction and usage of the additional input parameters *CosMax* and *TBOmin* might appear somewhat ad hoc, although they serve the legitimate purpose of identifying outliers and avoiding extreme solutions. Admittedly, this was a sort of 'quick fix' solution to the problem with the earlier model that behaved too conservatively. However, in real-life OR projects one often has to resort to such remedies because of lack of data, time or other resources, or simply because a more sophisticated approach would not be worth the effort. It is definitely a subject for further investigation, whether it would be worthwhile to deal with these matters in a theoretically more satisfactory way.

Finally, another issue that could be raised is whether the demand parameters (the parameters concerning order sizes and/or the parameters concerning time intervals) should be estimated from exponentially weighted historical data rather than through the moving average type procedure (by sliding a historic time window) that is suggested here. The latter is undoubtedly easy to apply in practice without further needs for parameter estimates. It is also justifiable in case of stationary demand. On the other hand, if demand is non-stationary, for example subject to drift, it would most likely be recommendable to refine the model by including some type of exponential smoothing procedure. This could be implemented along the lines suggested by Johnston and Boylan (1996) with EWMA adapted to Croston's decomposition approach. Such a procedure would still comply with the approach that has been used here to specify the demand process.

4. Inventory control

The inventory control policy used by the distribution centre at Danfoss' RA division is a base-stock policy with parameter (order-up-to level) S. The aim is to select S so that a satisfactory order fill rate (*OFR*) is achieved. The order fill rate is defined as the fraction of (the unknown) orders for which the whole order is available on time at the distribution centre. Before our common project, Danfoss used a standard heuristic method, essentially based on a cycle service level requirement, and the assumption that the lead-time demand distribution could be described by a normal distribution. However, this assumption was not fully satisfactory, because demands are often low frequency and individual customer orders differ in size. Furthermore, even if the normal distribution approximation did hold, as in the case of some high-volume SKUs, it was also clear that the service level requirement was not fully consistent with the preferred *OFR* service measure. An *OFR*-type measure is also what is used for *ex post* performance measurement of the distribution centre operation.

Our mathematical model is a base-stock model with the assumption that all unknown customer orders have agreed delivery lead time $DT_i = 0$. (As noted above, after the filtering procedure all advance-order information is suppressed.) Moreover, all replenishment orders have a constant lead time L. Unfilled demand is backlogged, and any order that cannot be filled immediately is assumed to be partially filled. The demand process is modelled as a compound renewal (Erlang) process with order size distribution X and time between order request distribution T, where the distributions of X and T have been specified and estimated as described in Section 3. Let D_L denote the aggregate demand during the replenishment lead time. Then D_L has the probability distribution (Cox, 1962)

$$P(D_L = x) = \begin{cases} e^{k\lambda L} \sum_{j=0}^{k-1} \frac{(k\lambda L)^j}{j!} & x = 0 \\ e^{k\lambda L} \sum_{m=1}^{x} P(X(m) = x) \sum_{j=0}^{k-1} \frac{(k\lambda L)^{j+mk}}{(j+mk)!} & x = 1, 2, \ldots \end{cases}$$

Note that in the case where the Erlang phase factor $k = 1$, this expression can be rewritten recursively; see Adelson (1966). The *OFR* service measure can now be determined in the following way. With probability $P(D_L = x)$ there will at the time of an order request be a net inventory of $S- x$ units. Therefore, the whole order will be filled with probability $P(X \leq S - x)$. These two probabilities are independent. Hence, the *OFR*, as a function of S, is given by

$$OFR(S) = \sum_{x=0}^{S-1} P(D_L = x) P(X \leq S - x)$$

For a given required level α of the service measure *OFR*, we then find the smallest value of *S* that satisfies $OFR(S) \geq \alpha$. Larsen and Thorstenson (2007) elaborate further on the *OFR* measure and specify in some detail how it is related to the standard (volume) fill rate and to some other service level concepts.

5. Evaluation

During the later phase of this project, in the Spring of 2005, two master's students worked at Danfoss to validate the model and facilitate the implementation process at the distribution centre. Their work is reported in a master's thesis (Bundgaard and Dahlgaard, 2005), which was supervised by two of the current authors. Numerical results reported in this section have been extracted from this thesis. It was also during this project phase that it was found necessary to make a generalization of the inventory control model, as described in Section 3. Thus, by *New Model 1* we refer to the model with a customer order arrival process following a Poisson process. In case the (delayed) negative binomial distribution is chosen, the parameters are estimated by fitting the empirical mean and variance to the distribution. *New Model 2* refers to the final version of our model. Moreover, *Old Model* refers to the tool used by Danfoss before the development of the two new inventory control models. The *Old Model* was essentially a series of more or less formalized rules of thumb. Therefore, today it is virtually impossible to reconstruct results using that model. However, in the Spring of 2005 it was still running.

Initially, a comparison was made between the *Old Model* and the *New Model 1* using data for *5138* SKUs. The required *OFR* was *0.98* for the so-called *A* items (*3134* SKUs) and for the remaining items, denoted *B* items (*2002* SKUs), the required *OFR* was *0.90*. Using these fill rate requirements *New Model 1* was run and the computed base-stock levels were compared with the current base-stock levels computed by the *Old Model*. The numbers of SKUs with base stocks lower, the same, or higher than the old model are shown in Table 1.

<Table 1 about here>

From Table 1 it could be concluded that using the base-stock levels obtained from *New Model 1* would increase the inventory investment for approximately 80% of SKUs.

However, this conclusion involves two caveats. First, are the realized inventory service levels resulting from the use of *Old Model* more modest than the stated requirements would suggest? Second, do the demand process assumptions underlying *New Model 1* fit sufficiently well to reality? As argued in the following, answers to both these questions are helpful in explaining the results in Table 1.

In order to explore these issues further a VBA tool was developed to perform a so-called "rear-view mirror" analysis. The idea is simply to reconstruct the inventory records using a given base-stock level and the historic demand information (customer order and delivery times, as well as order sizes) during a given time window. The analysis is illustrated graphically in Figure 1 for two SKUs.

<Figure 1 about here>

Two different performance measures were analyzed, the realized *OFR*, measured as the fraction of customer orders satisfied without stockouts, and the realized *Dead Stock Fraction* (*DSF*). The latter was introduced because the cases with *OFR = 1* can cover various degrees of over performance, i.e. overstocking. The DSF was measured as

$DSF = \max\{0, MinNetInv/S\}$,

where *MinNetInv* is the minimum net inventory level observed in the "rear-view mirror" analysis. From Figure 1 it can be observed that SKU 003N2119 has *OFR = 1* and *DSF* = *0.67* (*S = 120* and *MinNetInv = 80*), while SKU 011L1103 has *OFR < 1* and *DSF* = 0 (as *MinNetInv < 0*).

To answer the first question above, the realized *OFR* levels were computed using the base-stock levels suggested by the *Old Model*. This resulted in an average realized *OFR* of *0.79* for the *A* items and *0.65* for the *B* items. Hence, the analysis revealed a considerable discrepancy between statements about required service levels and the realized service levels. The corresponding realized *OFR* values when using the base-stock levels suggested by *New Model 1* (using the required *OFR* levels of *0.98* and *0.90*) were *0.98* and *0.92*, for the A and B items respectively. (Obviously, a weakness with this type of analysis is that these same data were used for estimation and evaluation; This contributes to the close correspondence between required and realized *OFR* levels.) Thus, it became clear that the comparisons in Table 1 are not fair. A rerun of *New Model 1* using the resulting *OFR* levels from the *Old Model*, i.e. a required *OFR* of *0.79* for the *A* items and *0.65* for the *B* items, predicted a 6% decrease in the overall inventory investment.

Hence, it could be concluded that when compared to the *Old Mode*l, *New Model 1* is more transparent (in the sense that its decision rules are explicit) and more consistent (in the sense that its outcome corresponded more closely with the expected requirements). Moreover, if applied on an equitable basis, *New Model 1* produced results that were slightly better in terms of the inventory levels. However, there was still the phenomenon of a certain level of overperformance for SKU 003N2119 in Figure 1. The same phenomenon appeared for several other SKUs as well. This provided the main motivation for creating the *New Model 2* with a renewal rather than a Poisson demand inter-arrival process as in *New Model 1*.

In order to analyze the relative performance of these two models the SKUs were grouped into three categories of "over- and underperformers" with respect to the *OFR* and *DSF* measures. Performance group 1 contains SKUs with less than 5% deviation from the required *OFR*, whereas groups 2 and 3 contain SKUs with larger deviations. For overperformers with *OFR* = 1,00 the *DSF* measure is used to differentiate between sub-groups. The classification is shown in Table 2.

<Table 2 about here>

A comparison between *New Model 1* and *New Model 2* on a new data set consisting of 3578 *A* items (using a required *OFR* of *0.98*) and 2684 *B* items (using a required *OFR* of *0.90*), provided the results presented in Tables 3 and 4.

<Tables 3 and 4 about here>

It can be observed that using *New Model 2* significantly reduces the degree of over-performance, i.e. overstocking (a main concern at Danfoss), but at the price of increased underperformance. Table 3 shows that, for the *A* items, there is a considerable reduction in the number of SKUs in subgroup O3, while subgroups U2 and U3 have an increase. However, more SKUs are placed in subgroups *U1* and *O1* than when using *New Model 1*.

For the *B* items in Table 4, the results are more mixed from using *New Model 2*. While the overperformance is reduced, there is a significant increase in the number of underperforming SKUs, i.e. items providing an inventory service level below the stated requirement.

6. Implementation and application

In this section we summarize the computational procedure for updating the inventory control system parameters and its applications. Data are first extracted from the Business Warehouse system at Danfoss' RA division. Next, the filtering procedure (see Section 3) is conducted. Thereafter the average order size (*EmpM*), order size variance (*EmpVar*), average number of orders per day (λ) , minimum time between orders (*TBOmin*), and customer order size maximum (*CosMax*) are estimated and inserted as inputs in a spreadsheet. The inputs for the lead times (L) and the required OFR service levels (α) are also inserted. Data for each SKU are in separate rows. The computational procedure to find order-up-to levels S is programmed as a macro in Visual Basic for Excel (VBA). The macro reads input data for one SKU at a time. It then estimates (or forecasts) the demand process, i.e. it specifies the distributions of X, the customer order size, and T, the time between customer orders. Finally, it carries out repeated computations of $OFR(S)$ for incremental values of S until $OFR(S) \geq \alpha$. For an illustration of the computational procedure see Tables 5 and 6, where Table 5 contains the inputs, while the outputs are shown in Table 6. For comparative purposes, Table 6 also contains order-up-to levels S_{VFR} , and actual volume fill rates, *VFR* resulting from using α as the required volume fill rate. Note that S_{OFR} can take lower as well as higher values than S_{VFR} . It depends on the chosen customer order size distribution. Further analytical and numerical comparisons of the order-up-to levels and service levels can be found in Larsen and Thorstenson (2007).

<Tables 5 and 6 about here>

The VBA macro can compute base-stock levels for about *4000* SKUs in approximately one hour on a standard PC. Because the computational procedure for parameter updating is only intended to be used as an operational tool, say, once a month or once a quarter, this computational time does not appear to be an issue of major concern.

Regarding interpretation of the output from the computation, it is important to make users of the control tool aware of the fact that the computed service levels are expected values, corresponding to what would be observed in the long run. When performance is measured periodically on a short-term basis, the actual service levels obtained may of course differ considerably from the computed expectations for a specific SKU, especially for low frequency SKUs. This effect may be diminished, if the service levels are measured across groups of SKUs.

The analytical model is obviously intended as an operational tool, which can provide management with guidelines for setting base-stock levels appropriately. However, the model can also be applied as a tactical tool, conducting various sensitivity analyses, e.g. exploring the effects of reducing the variability of the unknown customer orders, increasing the fraction of known orders, reducing lead times, etc. Furthermore, it may be helpful in the reverse process of identifying required parameters to reach certain inventory-related performance goals. The results of such analyses might be useful as a guide for ranking different demand planning activities that Danfoss' RA division is considering. Note though, that when the model is used as a simulation tool for these purposes, the computational time may become an issue. A standard way to resolve this difficulty would be to work with a representative sample of the SKUs rather than with the complete set.

The distinction between operational and tactical purposes underlines the phrase: *It is often better to prevent than to cure*. Solely focusing on operational issues represents the cure. Having a tool which can support the analysis of future possible scenarios also qualifies it to be a vital part of the preventive effects obtained from redesigning business processes.

7. Conclusions and discussion of future work

This paper is concerned with the inventory system of a large spare parts distribution centre. We have presented the development and design of a model for demand specification and inventory control, as well as discussed issues related to its validation and application. A focal point of the model has been how to fit empirical demand estimates (forecasts) to a versatile demand process specified by a compound renewal process. This process is subsequently used as the input for minimization of the inventory levels subject to an *order fill rate* (*OFR*) constraint.

As in any OR-related project, the system dealt with has to be delineated, modelling assumptions have to be imposed and compromises have to be struck between what is desirable from a theoretical point of view and what is possible when considering the practical realities. The acid test for any practicable decision support tool for planning and control is whether it can contribute to improved business performance. Obviously, there is room for further improvements of the model. For example, at the present stage of development, there are no elements of coordination included either between the various SKUs or between the different tiers of the supply chain. However, we will specifically comment here on two other relevant issues, namely on the possible incorporation of advance order information and on alternative forecast (estimation) methods that could be better suited to cope with non-stationarities such as seasonality, drift, and trends.

As commented previously (Section 3), after completing the filtering process all other advance order information is suppressed. It is, however, possible to deduce mathematically an expression for *OFR(S)*, under the assumption that orders are served according to the (future) date when they are requested (so that customers who accept a later agreed delivery lead time could also be rewarded). However, this requires information about the distribution of advance orders. Related to this issue is also the structure of the optimal inventory control policy. In combination with the renewal demand process it might be of particular interest to consider some kind of delayed replenishment ordering policy rather than the pure base-stock policy, which is not the optimal policy in general for this case, as noted in Section 2.

With any forecast method it is a prerequisite to have access to valid data. A primary concern then is that of scarcity of data, which is typical for many spare parts with low frequency demands. In case of SKUs with only few data, an option could be to group items with similar characteristics in order to provide more robust forecasts. Evidently, the problem is first how to aggregate data and then how to decompose the (aggregate) base-stock level into base-stock levels for each separate SKU. Another way to cope with few data is to use an empirical distribution in combination with statistical bootstrapping. It could be an interesting subject for future research to make comparisons between our method and these alternative approaches.

The moving average approach of updating basic empirical statistics that are used as input in our estimation procedures is easy to use and justifiable under the assumption of stationarity of demand. However, it could be worth challenging this approach by comparing it to exponential smoothing procedures, as these are known to provide better forecasts in general in the presence of non-stationarities. Indeed, for some SKUs at the distribution centre it is acknowledged that there are elements of seasonality or trend in the demands. For those SKUs the forecast estimates of our model will lag behind, as there is no provision for such elements in the model other than frequent parameter updates. Most likely, it is possible to construct an improved version of our model to, at least partially, incorporate these elements in a more sophisticated way. Developments could be based on adopting some of the decomposition techniques presented in Silver et al (1998; Ch 4). Clearly, though, improvements will come at the expense of requirements for further inputs and computations. As a final comment, it should be noted that irrespective of the forecast method used, the general principles and procedures for the model specified in this paper are still applicable.

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Tables and Figures:

Base-stock level ordering	*S*(*New Model 1*) < *S*(*Old Model*)	S(*New Model 1*) = S(*Old Model*)	S(*New Model 1*) > S(*Old Model*)
SKU Count (Total: 5138)	822	192	4124

Table 1 Number of SKUs with *New Model 1* base-stock levels less than, equal to, or greater than with the *Old Model*.

Performance group	Sub-group	*A* items	*B* items
1	U1	*OFR*: [0,93; 0,98[	*OFR*: [0,85; 0,90[
	O1	*OFR*: [0,98; 1,00[or *DSF*:]0; 0,03]	*OFR*: [0,90; 0,95]
2	U2	*OFR*: [0,80; 0,93[	*OFR*: [0,70; 0,85[
	O2	*DSF*:]0,03; 0,20]	*OFR*:]0,95; 1[or *DSF*:]0,00; 0,10]
3	U3	*OFR*: [0; 0,80[	*OFR*: [0; 0,70[
	O3	*DSF*:]0,20; ∞[	*DSF*:]0,10; ∞[
	Required *OFR*	0.98	0.90

Table 2 Classification of over- and underperformance groups.

New Model 1:

	Underperformers			Overperformers			
	U3	U2	U1	O1	O2	O3	
SKU Count	30	166	633	743	771	1.235	3.578
%	0,8%	4,6%	17,7%	20,8%	21,5%	34,5%	100,0%

New Model 2:

	Underperformers			Overperformers			
	U3	U2	U1	O1	O2	O3	
SKU Count	329	628	763	935	660	263	3.578
%	9,2%	17,6%	21,3%	26,1%	18,4%	7,4%	100,0%

Table 3 Comparison between *New Model 1* and *New Model 2* for *A* products.

New Model 1:

	Underperformers			Overperformers			
	U3	U2	U1	O1	O2	O3	
SKU Count	51	295	392	499	825	622	2.684
%	1,9%	11,0%	14,6%	18,6%	30,7%	23,2%	100,0%

New Model 2:

	Underperformers			Overperformers			
	U3	U2	U1	O1	O2	O3	
SKU Count	410	637	420	424	695	98	2.684
%	15,3%	23,7%	15,6%	15,8%	25,9%	3,7%	100,0%

Table 4 Comparison between *New Model 1* and *New Model 2* for *B* products.

SKU	**Average order size** (*EmpM*)	**Variance order size** (*EmpVar*)	**Average no. of orders per day** (λ)	**Min. time between orders** (*TBOmin*)	**Max. customer order size** (*CosMax*)	**Lead time** (*L*)	**Required OFR** (α)
003N2107	15.500	0.500	0.008	79	16	13	0.98
003N2113	24.000	375.000	0.042	0	60	6	0.90
003N2114	11.444	280.540	0.138	0	60	13	0.98
003N2119	38.429	133.187	0.054	2	57	13	0.98
003N2125	3.067	10.781	0.058	2	14	13	0.98
003N2128	25.000	0.000	0.004	50	25	6	0.90
003N2132	2.670	11.611	1.338	0	30	6	0.98
003N2162	3.816	35.538	4.442	0	50	6	0.98
003N2164	16.941	163.059	0.065	1	60	6	0.90

Table 5 Input data for the VBA macro to compute inventory control parameters (time unit is days). Note that the parameter *CosMax* is only used in case the customer order-size distribution is chosen to be negative binomial.

	Erlang phase	Customer order size distribution characteristics			Output for inventory control			
SKU	k	Type	Form param.	Prob. param.	S_{OFR}	Actual *OFR*	S_{VFR}	Actual *VFR*
003N2107	9	Binomial	17.000	0.9655	17	1.000	17	1.000
003N2113	1	NegBin	4.424	0.8387	54	0.901	46	0.905
003N2114	1	NegBin	1.661	0.8628	87	0.981	84	0.980
003N2119	3	NegBin	79.911	0.3190	103	0.980	89	0.980
003N2125	3	NegBin	0.851	0.7083	15	0.985	15	0.984
003N2128	4	Binomial	25.000	1.0000	25	1.000	23	0.920
003N2132	1	NegBin	0.280	0.8562	57	0.981	59	0.981
003N2162	1	NegBin	0.242	0.9207	195	0.981	199	0.980
003N2164	2	NegBin	1.727	0.9022	41	0.902	37	0.902

Table 6 Output from the macro given the input in Table 5.

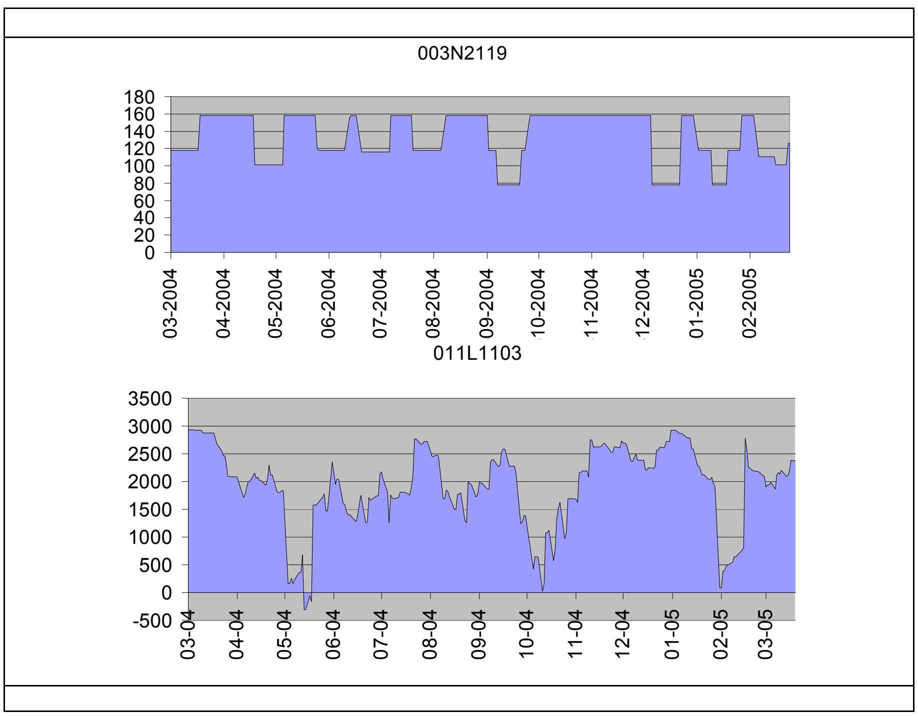

Figure 1: Examples of two "rear-view mirror" analyses in graphical format for SKUs 003N2119 and 011L1103. Time is on the horizontal axis and the (net) inventory is on the vertical axis.

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